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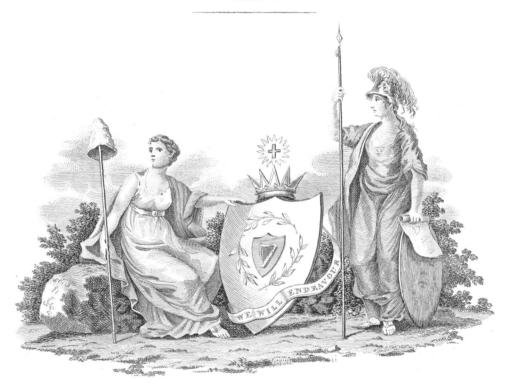
# **TRANSACTIONS**

OF THE

# ROYAL IRISH ACADEMY,

VOL. XVI.

PART I.



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R. GRAISBERRY,

PRINTER TO THE ROYAL IRISH ACADEMY.

1830.

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# SCIENCE.

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FUNDAMENTAL formula of optical systems ordinary and extraordinary, or equation of the characteristic function;

$$V = \int v ds$$
;  $\partial V = \frac{\partial v}{\partial x} \partial x + \frac{\partial v}{\partial s} \partial y + \frac{\partial v}{\partial y} \partial z$ ; ... numbers 1, 2, 3.

Other characteristic function, for systems of straight rays;

$$W + V = z \frac{\partial v}{\partial \alpha} + y \frac{\partial v}{\partial \beta} + z \frac{\partial v}{\partial \gamma}; \ \partial W = x \partial \frac{\partial v}{\partial \alpha} + y \partial \frac{\partial v}{\partial \beta} + z \partial \frac{\partial v}{\partial \gamma}; \quad \dots \quad 4.$$

Connexions between the partial differential coefficients of the two characteristic functions, and changes produced by reflexion or refraction, ordinary or extraordinary;

$$\partial \triangle V = \lambda \partial u; \ \triangle W = \lambda \left( z \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right); \quad ... \quad 5, 6, 7.$$

Connexions of the characteristic functions with the developable pencils, and with the caustic curves and surfaces;

$$e^{\delta} \frac{\partial V}{\partial x} = \delta' \frac{\partial v}{\partial \omega}, e^{\delta} \frac{\partial V}{\partial y} = \delta' \frac{\partial v}{\partial \beta}, e^{\delta} \frac{\partial V}{\partial z} = \delta' \frac{\partial v}{\partial \gamma};$$

$$\delta' \frac{\partial W}{\partial \omega} = \delta' \frac{\partial W'}{\partial \omega}, \delta' \frac{\partial W}{\partial \beta} = \delta' \frac{\partial W'}{\partial \beta}, \delta' \frac{\partial W}{\partial \gamma} = \delta' \frac{\partial W'}{\partial \gamma}; \dots 8, 9, 10.$$

On osculating focal systems; the foci of the extreme systems are contained upon the caustic surfaces, and their planes of osculation touch the developable pencils; the foci of the other osculating systems are inside or outside the interval of the extreme foci, according to the sign of a certain function v'', which does not change by transformation of coordinates; the distances of any one of these other foci from the two caustic surfaces are proportional to the squares of the sines of the angles which the corresponding plane of osculation makes with the tangent planes to the two developable pencils;

$$\mathcal{W} = \mathcal{V}W'; \frac{R - R_1}{R_2 - R} = \zeta \left(\frac{\sin.(\varphi - \varphi_1)}{\sin.(\varphi_2 - \varphi)}\right)^2; \quad \frac{\zeta}{v''} > 0; \dots 11, 12, 13.$$

Principal foci, and principal rays: the principal foci belong to those osculating systems, for which the osculation is most complete; they are the only points in

which the caustic surfaces intersect, when the function v'' is positive; this condition is satisfied for ordinary systems, and for the systems produced by crystals with one axis:

$$\frac{\partial^{2}W}{\partial \alpha^{2}} = \frac{\delta^{2}W'}{\partial \alpha^{2}}, \quad \frac{\partial^{2}W}{\partial \beta^{2}} = \frac{\partial^{2}W'}{\partial \beta^{2}}, \quad \frac{\partial^{2}W}{\partial \gamma^{2}} = \frac{\partial^{2}W'}{\partial \gamma^{2}}; \quad \dots \quad 14.$$

On osculating spheroids, and surfaces of constant action; the extreme spheroids have their centres upon the caustic surfaces, and their planes of osculation touch the developable pencils; the centre of the osculating spheroid coincides with the focus of the osculating focal reflector or refractor, when the point of contact and the plane of osculation are the same; the principal foci are centres of spheroids which have complete contact of the second order with the surfaces of constant action; they are also foci of osculating focal reflectors or refractors which have contact of the same kind with the last given reflector or refractor;

$$\partial_{z} V = \partial_{z} V'; \quad \frac{\xi_{1}^{-1} - \xi_{1}^{-1}}{\xi_{1}^{-1} - \xi_{2}^{-1}} = \zeta \left( \frac{\sin (\psi - \psi_{1})}{\sin (\psi_{2} - \psi_{1})} \right)^{2}; \quad \dots \quad 15, 16$$

On foci by projection and virtual foci; the planes of extreme projection coincide with the planes of extreme virtual foci; these planes are perpendicular to each other, and furnish in general a pair of natural coordinates; the tangent planes to the developable pencils are symmetrically situated with respect to these natural coordinate planes, and coincide with them when the system is rectangular; this latter condition is satisfied in ordinary systems, and the osculating foci of such systems coincide with the foci by projection;

$$p = -\frac{\partial x'^2 + \partial y'^2 + \partial z'^2}{\partial \omega \partial x' + \partial \beta \partial y' + \partial \gamma \partial z'}; \quad \frac{1}{p} = \frac{(\cos \cdot \Pi)^2}{p_{\cdot}} + \frac{(\sin \cdot \Pi)^2}{p_{\omega}};$$

$$r = -\frac{\partial \omega \partial x_{\cdot} + \partial \beta \partial y_{\cdot} + \partial \gamma \partial z_{\cdot}}{\partial \omega^2 + \partial \beta^2 + \partial \gamma^2}; \quad r = r_1 (\cos \cdot \omega)^2 + r_2 (\sin \cdot \omega)^2; \quad ... \quad 17, 18.$$

Theory of lateral aberrations; aberration from a principal focus; elliptic or hyperbolic lines of uniform condensation; natural axes of the system;

$$\Delta x = [\delta x] + \frac{1}{2} [\delta^{n}x] + \frac{1}{2 \cdot 3} [\delta^{n}x] + \&c.$$

$$M^{n} = \left\{ \frac{\delta^{n}W}{\delta a^{3}} \frac{\delta^{n}W}{\delta a \delta \beta^{n}} - \left(\frac{\delta^{n}W}{\delta x^{2} \delta \beta}\right)^{2} \right\} a^{n} + \left\{ \frac{\delta^{n}W}{\delta a^{2} \delta \beta} \frac{\delta^{n}W}{\delta \beta^{n}} - \left(\frac{\delta^{n}W}{\delta a \delta \beta^{n}}\right)^{2} \right\} b^{n};$$

$$\frac{\delta^{n}W}{\delta a^{n}} \frac{\delta^{n}W}{\delta a^{n}} = \frac{\delta^{n}W}{\delta a^{n} \delta \beta} \frac{\delta^{n}W}{\delta a \delta \beta^{n}};$$
19, 20.

# POLITE LITERATURE.

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